Estimation Theory, Final exam 20.10.2017, Marko Vauhkonen

1

Joint density function is given

$$f(x,y) = \begin{cases} \frac{3}{2}(x^2+y^2), & \text{when } 0<x<1, \quad 0<y<1 \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

Calculate $E\{x|y=\frac{1}{2}\}$ and draw $f(x|y=\frac{1}{2})$.

2

Prove that $f(z|\theta) = f_v(z-h(\theta))$, when θ and v are independent

3

a) Calculate $\hat{\theta}_{\text{LMMS}}$ when observation model is

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \theta + v \tag{2}$$

Assume $\eta_v = \eta_\theta = 0$ and variances to be σ_θ^2 and σ_v^2
b) What happens when $\sigma_\theta^2 \to \infty$?

4

a) It's known that in Bayes cost method

$$B(\hat{\theta}|z) = E\left\{C(\theta,\hat{\theta})|z\right\}. \tag{3}$$

Prove that

$$B(\hat{\theta}|z) = \text{trace } C_{\theta|z} + ||\hat{\theta} - \eta_{\theta|z}||^2 \tag{4}$$

for MS estimate.
b) Based on a) what is $\hat{\theta}_{MS}$ and why?

5

Density functios are

$$f(\theta) = \begin{cases} 1 & \text{when } 0<\theta<1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

and

$$f_v(v) = \begin{cases} \frac{1}{2}v & \text{when } 0<v<2 \\ 0 & \text{otherwise} \end{cases} . \tag{6}$$

One observation was made: z $= \frac{5}{2}$. Calculate $\hat{\theta}_{\text{MS}}$, $\hat{\theta}_{\text{ML}}$ and $\hat{\theta}_{\text{MAP}}$ and draw corrensponding figures.