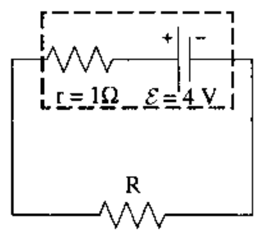
Fysiikan Peruskurssi 2 (FP2) 1.välitentti 5.12.2005 I

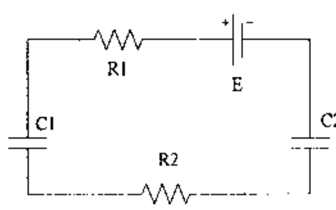
1. Sähköisesti neutraalin johdepallokuoren sisäsäde on r_a= 3 cm ja ulkosäde r_b = 10 cm. Pallon keskipisteeseen asetetaan +5 nC:n pistevaraus. Soveltaen Gaussin lakia määritä sähkökenttä Alueissa r = 2 cm, r = 5 cm ja r = 15 cm. (10p)

2. Varaus Q on jakaantunut tasaisesti a –säteiseen renkaaseen. Olkoon renkaan keskipiste origossa ja rengas itse y-z –tasossa. Renkaan akselina on x –akseli.
   a. Määritä potentiaali renkaan akselilla, etäisyydellä x renkaan keskipisteestä.(5p)
   b. Tukeutuen potentiaalin gradienttiin, määritä sähkökenttä renkaan akselilla etäisyydellä x renkaan keskipisteestä.(5p)

3. Yksinkertaisessa virtapiirissä on jännitelähde, jonka sähkömotorinen voima $\mathcal{E}$ = 4 V ja sisäinen resistanssi r = 1Ω. Piirissä on ulkoinen vastus R. Millä vastuksen R arvolla vastuksessa kulutettu teho P_R on suurimmillaan?(10p)



4. Alla olevassa kuvassa on RC –piiri, jossa R_1 = 10Ω, R_2 = 300Ω, E = 100V, C_1=C_2= 500pF. Piiriä lähdetään lataamaan ajanhetkellä t = 0 , jolloin kondensaattorit ovat aluksi varauksettomat. Määritä jännitteet kondensaattoreiden yli ajanhetkellä, jolloin piirissä kulkeva virta on 70% maksimivirrasta.(10p)



5. Selitä lyhyesti (käytä lisäksi mahdollisia kuvia ja kaavoja):
   a. Mihin perustuu kondensaattorien sarjaankytkennän kaava $\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+...+\frac{1}{C_n}$? (5p)
   b. Mihin sähköopin periaatteisiin perustuu Kirchoffin säännöt (solmupistesääntö $\sum I = 0$ ja silmukkasääntö $\sum V = 0$)?(5p)

Tentissä saa olla mukana kirjoitusvälineet, laskin, Maolin ja lukion taulukot, sekä Tammer tekniikan kaavastot. Tämän tehtäväpaperin liitteineen saa ottaa mukaan tenttitilaisuudesta.

Vector Identities

$$(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} = (\mathbf{B}\times\mathbf{C})\cdot\mathbf{A} = (\mathbf{C}\times\mathbf{A})\cdot\mathbf{B}$$

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\cdot\mathbf{C})\mathbf{B} - (\mathbf{A}\cdot\mathbf{B})\mathbf{C}$$

$$\nabla\cdot(V\mathbf{A}) = \mathbf{A}\cdot\nabla V + V\nabla\cdot\mathbf{A}$$

$$\nabla\times(V\mathbf{A}) = \nabla V\times\mathbf{A} + V\nabla\times\mathbf{A}$$

$$\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot\nabla\times\mathbf{A} - \mathbf{A}\cdot\nabla\times\mathbf{B}$$

$$\nabla(\mathbf{A}\cdot\mathbf{B}) = (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A} + \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A})$$

$$\nabla\cdot\nabla = \nabla^2$$

$$\nabla\cdot(\nabla\times\mathbf{A}) = 0$$

$$\nabla\times(\nabla V) = 0$$

$$\nabla\times\nabla\times\mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$$

$$\int_V(\nabla\cdot\mathbf{A})dv = \oint_S \mathbf{A}\cdot d\mathbf{s}$$

$$\int_S(\nabla\times\mathbf{A})\cdot d\mathbf{s} = \oint_C \mathbf{A}\cdot d\boldsymbol{\ell}$$

$$\int_V \nabla\times\mathbf{F}dv = -\oint_S \mathbf{F}\times d\mathbf{s}$$

Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates (x, y, z)

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{a}}_x + \frac{\partial f}{\partial y}\hat{\mathbf{a}}_y + \frac{\partial f}{\partial z}\hat{\mathbf{a}}_z$$

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{\mathbf{a}}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\hat{\mathbf{a}}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{\mathbf{a}}_z$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical Coordinates (ρ, ϕ, z)

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\mathbf{a}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\mathbf{a}}_\phi + \frac{\partial f}{\partial z}\hat{\mathbf{a}}_z$$

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\rho)\right] + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi} - \frac{\partial A_\phi}{\partial z}\right]\hat{\mathbf{a}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\hat{\mathbf{a}}_\phi + \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]\hat{\mathbf{a}}_z$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates (r, θ, ϕ)

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathbf{a}}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\mathbf{a}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\mathbf{a}}_\phi$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\left[\frac{\partial}{\partial r}(r^2 A_r)\right] + \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\theta\sin\theta)\right] + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi\sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right]\hat{\mathbf{a}}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}(rA_\phi)\right]\hat{\mathbf{a}}_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\mathbf{a}}_\phi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$$

Electrostatic field equations

Coulomb's law:	$\vec{\mathbf{F}} = q\vec{\mathbf{E}}$		
Electric field:	$\vec{\mathbf{E}} = \dfrac{Q\vec{\mathbf{a}}_R}{4\pi\epsilon R^2}$	or	$\vec{\mathbf{E}} = \dfrac{1}{4\pi\epsilon}\displaystyle\int_v \dfrac{\rho\vec{\mathbf{a}}_R}{R^2}\,dv$
Gauss's law:	$\nabla\cdot\vec{\mathbf{D}} = \rho$	or	$\oint_s \vec{\mathbf{D}}\cdot\vec{ds} = Q$
Conservative $\vec{\mathbf{E}}$ field:	$\nabla\times\vec{\mathbf{E}} = 0$	or	$\oint_c \vec{\mathbf{E}}\cdot\vec{d\ell} = 0$
Potential function:	$\vec{\mathbf{E}} = -\nabla V$	or	$V_{ba} = -\displaystyle\int_a^b \vec{\mathbf{E}}\cdot\vec{d\ell}$
Poisson's equation:	$\nabla^2 V = -\dfrac{\rho}{\epsilon}$		
Laplace's equation:	$\nabla^2 V = 0$		
Energy density:	$w_e = \frac{1}{2}\vec{\mathbf{D}}\cdot\vec{\mathbf{E}}$		
Constitutive relationship:	$\vec{\mathbf{D}} = \epsilon\vec{\mathbf{E}}$		
Ohm's law:	$\vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$		

Magnetostatic field equations

Force equation:	$\vec{\mathbf{F}} = q\vec{\mathbf{u}}\times\vec{\mathbf{B}}$	or	$\vec{d\mathbf{F}} = I\vec{d\ell}\times\vec{\mathbf{B}}$
Biot–Savart law:	$\vec{d\mathbf{B}} = \dfrac{\mu}{4\pi}\dfrac{I\vec{d\ell}\times\vec{\mathbf{a}}_r}{r^2}$		
Ampère's law:	$\nabla\times\vec{\mathbf{H}} = \vec{\mathbf{J}}$	or	$\oint_c \vec{\mathbf{H}}\cdot\vec{d\ell} = I$
Gauss's law:	$\nabla\cdot\vec{\mathbf{B}} = 0$	or	$\oint_s \vec{\mathbf{B}}\cdot\vec{ds} = 0$
Magnetic vector potential:	$\vec{\mathbf{B}} = \nabla\times\vec{\mathbf{A}}$	or	$\vec{\mathbf{A}} = \dfrac{\mu}{4\pi}\displaystyle\int_c \dfrac{I\vec{d\ell}}{r}$
Magnetic flux:	$\Phi = \displaystyle\int_s \vec{\mathbf{B}}\cdot\vec{ds}$	or	$\Phi = \oint_c \vec{\mathbf{A}}\cdot\vec{d\ell}$
Magnetic energy:	$w_m = \frac{1}{2}\vec{\mathbf{B}}\cdot\vec{\mathbf{H}}$		
Poisson's equation:	$\nabla^2\vec{\mathbf{A}} = -\mu\vec{\mathbf{J}}$		
Constitutive relationship:	$\vec{\mathbf{B}} = \mu\vec{\mathbf{H}}$		

$$\nabla\times\vec{\mathbf{E}} = -\frac{\partial\vec{\mathbf{B}}}{\partial t} \Rightarrow \oint_c \vec{\mathbf{E}}\cdot\vec{d\ell} = -\int_s \frac{\partial\vec{\mathbf{B}}}{\partial t}\cdot\vec{ds}$$

$$\nabla\times\vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial\vec{\mathbf{D}}}{\partial t} \Rightarrow \oint_c \vec{\mathbf{H}}\cdot\vec{d\ell} = \int_s \vec{\mathbf{J}}\cdot\vec{ds} + \int_s \frac{\partial\vec{\mathbf{D}}}{\partial t}\cdot\vec{ds}$$

$$\nabla\cdot\vec{\mathbf{D}} = \rho_v \Rightarrow \oint_s \vec{\mathbf{D}}\cdot\vec{ds} = \int_v \rho_v\,dv$$

$$\nabla\cdot\vec{\mathbf{B}} = 0 \Rightarrow \oint_s \vec{\mathbf{B}}\cdot\vec{ds} = 0$$

$$E_{t1} = E_{t2}$$
$$H_{t1} - H_{t2} = J_s$$
$$B_{n1} = B_{n2}$$
$$D_{n1} - D_{n2} = \rho_s$$
$$J_{n1} = J_{n2}$$
$$\frac{J_{t1}}{\sigma_1} = \frac{J_{t2}}{\sigma_2}$$

$$\vec{\mathbf{a}}_n\times(\vec{\mathbf{E}}_1 - \vec{\mathbf{E}}_2) = 0$$
$$\vec{\mathbf{a}}_n\times(\vec{\mathbf{H}}_1 - \vec{\mathbf{H}}_2) = \vec{\mathbf{J}}_s$$
$$\vec{\mathbf{a}}_n\cdot(\vec{\mathbf{B}}_1 - \vec{\mathbf{B}}_2) = 0$$
$$\vec{\mathbf{a}}_n\cdot(\vec{\mathbf{D}}_1 - \vec{\mathbf{D}}_2) = \rho_s$$
$$\vec{\mathbf{a}}_n\cdot(\vec{\mathbf{J}}_1 - \vec{\mathbf{J}}_2) = 0$$
$$\vec{\mathbf{a}}_n\times\left[\frac{\vec{\mathbf{J}}_1}{c_1} - \frac{\vec{\mathbf{J}}_2}{\sigma_2}\right] = 0$$

$$\tilde{\mathbf{J}}_t = \tilde{\mathbf{J}}_c + \tilde{\mathbf{J}}_d$$
$$\tilde{\mathbf{J}}_d = j\omega\epsilon\tilde{\mathbf{E}}$$
$$\tilde{\mathbf{J}}_c = \sigma\tilde{\mathbf{E}}$$

$$\tan\delta = \frac{\sigma}{\omega\epsilon}$$

$$\nabla\times\tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{B}} \Rightarrow \oint_c \tilde{\mathbf{E}}\cdot\vec{d\ell} = -j\omega\int_s \tilde{\mathbf{B}}\cdot\vec{ds}$$

$$\nabla\times\tilde{\mathbf{H}} = \tilde{\mathbf{J}} + j\omega\tilde{\mathbf{D}} \Rightarrow \oint_c \tilde{\mathbf{H}}\cdot\vec{d\ell} = \int_s \tilde{\mathbf{J}}\cdot\vec{ds} + j\omega\int_s \tilde{\mathbf{D}}\cdot\vec{ds}$$

$$\nabla\cdot\tilde{\mathbf{D}} = \tilde{\rho}_v \Rightarrow \oint_s \tilde{\mathbf{D}}\cdot\vec{ds} = \int_v \tilde{\rho}_v\,dv$$

$$\nabla\cdot\tilde{\mathbf{B}} = 0 \Rightarrow \oint_s \tilde{\mathbf{B}}\cdot\vec{ds} = 0$$

$$\nabla\cdot\tilde{\mathbf{J}} = -j\omega\tilde{\rho}_v \Rightarrow \oint_s \tilde{\mathbf{J}}\cdot\vec{ds} = -j\omega\int_v \tilde{\rho}_v\,dv$$

Skaalaustekijät, siirtymäalkiot ja yksikkövektorit

	Karteesinen	Sylinteri	Pallo
h_1	1	1	1
h_2	1	ρ	r
h_3	1	1	$r\sin\theta$
du_1	dx	$d\rho$	dr
du_2	dy	$d\phi$	$d\theta$
du_3	dz	dz	$d\phi$
$\hat{e}_1$	$\hat{i}$	$\hat{e}_\rho$	$\hat{e}_r$
$\hat{e}_2$	$\hat{j}$	$\hat{e}_\phi$	$\hat{e}_\theta$
$\hat{e}_3$	$\hat{k}$	$\hat{e}_z$	$\hat{e}_\phi$

Kantavektorit

$\hat{\mathbf{e}}_\rho = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$

$\hat{\mathbf{e}}_\phi = -\sin\phi\hat{\mathbf{i}} + \cos\phi\hat{\mathbf{j}}$

$\hat{\mathbf{e}}_z = \hat{\mathbf{k}}$

$\hat{\mathbf{e}}_r = \sin\theta\cos\phi\hat{\mathbf{i}} + \sin\theta\sin\phi\hat{\mathbf{j}} + \cos\theta\hat{\mathbf{k}}$

$\hat{\mathbf{e}}_\theta = \cos\theta\cos\phi\hat{\mathbf{i}} + \cos\theta\sin\phi\hat{\mathbf{j}} - \sin\theta\hat{\mathbf{k}}$

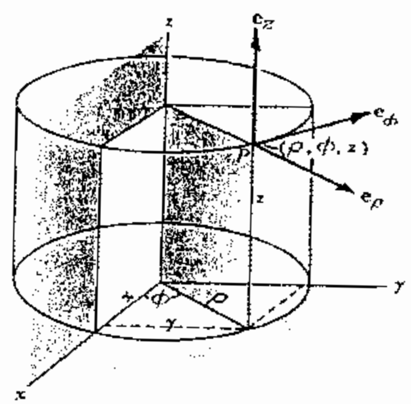
$\hat{\mathbf{e}}_\phi = -\sin\phi\hat{\mathbf{i}} + \cos\phi\hat{\mathbf{j}}$

Kaari: $d\bar{r} = h_1du_1\hat{e}_1 + h_2du_2\hat{e}_2 + h_3du_3\hat{e}_3$

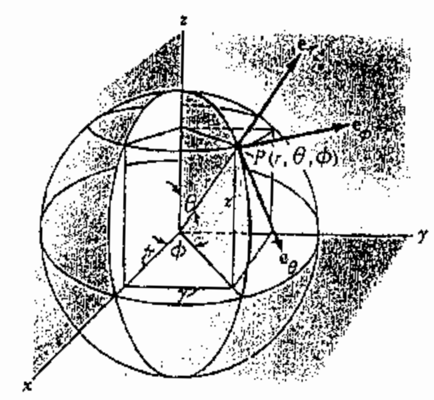
~~Pinta-ala yksikkövektorin $\hat{e}_1$ suunnalla :~~ $\cancel{d\bar{A} = h_2h_3du_2du_3\hat{e}_1}$

Tilavuuselementti: $dV = h_1h_2h_3du_1du_2du_3$

$0 < \phi < 2\pi,\qquad 0 < \theta < \pi,\qquad \rho \geq 0\qquad r \geq 0$



Sylinterikoordinaatisto



Pallokoordinaatisto

Markku Saarelainen
2003

Maxwellin yhtälöt